Problem Set 1: Waves, Light and Interference

1. A harmonic plane wave in a water pond is moving in a particular direction that we can call x-axis. The wave has an amplitude of $0.5\,m$ and a wavelength of $0.25\,m$ wheras its speed is $2\,m/s$. What is the angular frequency and angular wave number of this wave? How many waves fit in in a length of $10\,m$. Suppose at $t = 0$ there is a full peak at $x = 0$. What is the height of water at $x = 2\,m$ and $x = 2.1\,m$ at this time? What is the height at these same points at $t = 1\,s$?

2. Consider two waves that are superposing

$$f_1(x) = A\sin(kx,)$$

$$f_2 = A\sin(kx + \frac{\pi}{4})$$

.

What is the wavelength of the resultant wave? What is the amplitude of the resultant wave?

3. **Speakers in a gathering**

Two speakers are placed at a distance of 20 meters from each other. Consider the line joining the two speakers. Let us set up a coordinate system so that this line is the x-axis and speaker 1 is placed at $x = 0$. Then, the other speaker is placed at $x = 20m$. Both speakers are given same electric signal and they are producing static sound consisting of harmonic wave of frequency $1000Hz$.

a- Write down an expression for the sound wave from speaker 1 along the x-axis such that at $t = 0$ the wave starts as a pure sine wave from $x = 0$ with out any phase shift. Speed of sound is $340m/s$.

b- Now right down an expression for the wave from second speaker. Notice that the same feed is being given to the second speaker. Hence you can get the expression for second speaker by translating the first wave to the right by $20m$. How would you do that. Also, this wave is moving in opposite direction, but you can take care of that by reversing the relative sign of x and t terms in the sine. Also, be careful that the wave starts in the ascending fashion to the left. How will you make that sure?

c- Now let us look at this wave at a fixed time, say, $t = 0$. On what spots on the line will you hear the loudest sound? On what spots will you hear the faintest sound?

4. **FM Radios**

a- While listening to your favorite FM channel in a parked car, you must have noticed that sometimes the reception is affected if you displace your car slightly. Keeping in mind the phenomena of inteference and possibility of reflection of radio waves from nearby objects, explain why this could be happening?

b- For a channel broadcasted at a frequency of 100 Mhz, by what minimum distance you must move your car (in an appropriate direction) for the signal to become pronounced, if you start from a spot of diminished signal?

5. **Dodging the Police**

Police radar speed guns basically work by reflecting electromagnetic waves off the car's surface. Suppose you want to drive real fast on the motor way and somehow you have tracked exactly what the frequency of police speed radar is (usually it is somewhere around 20 GHz). Can you think of a design feature that you can include on the car's surface so that your car becomes invisible to the police radar.

6. **Rainbow on spilled oil**

Suppose there is a thin film of oil spread over some water. The film is only 4.8 micro meter thick. We look at this film perpendicularly from the top. Which color of the light will be seen brightest? Which color will be most suppressed? For simplicity, consider only three colors; Red with a wavelength of 400 nano meter, Green with a wavelength of 520 nanometer and Blue with a wavelength of 640 nano meter. Recall that the light rays will be reflected from top and bottom of the film and those two rays interfere.

7. **Measuring Wavelengths with an Interferometer**

Consider the interferometer as shown in figure. Two monochromatic light rays that are initially coming from same source (and hence being initially in phase) travel the two different paths and finally we look at them on the screen and find an interference pattern. Suppose we change the position of one of the mirrors (mirror A in the figure) by 0.0125 mm and find that the bright and dark fringes change into each other 100 times during this process. What must have been the frequency of the light ray?

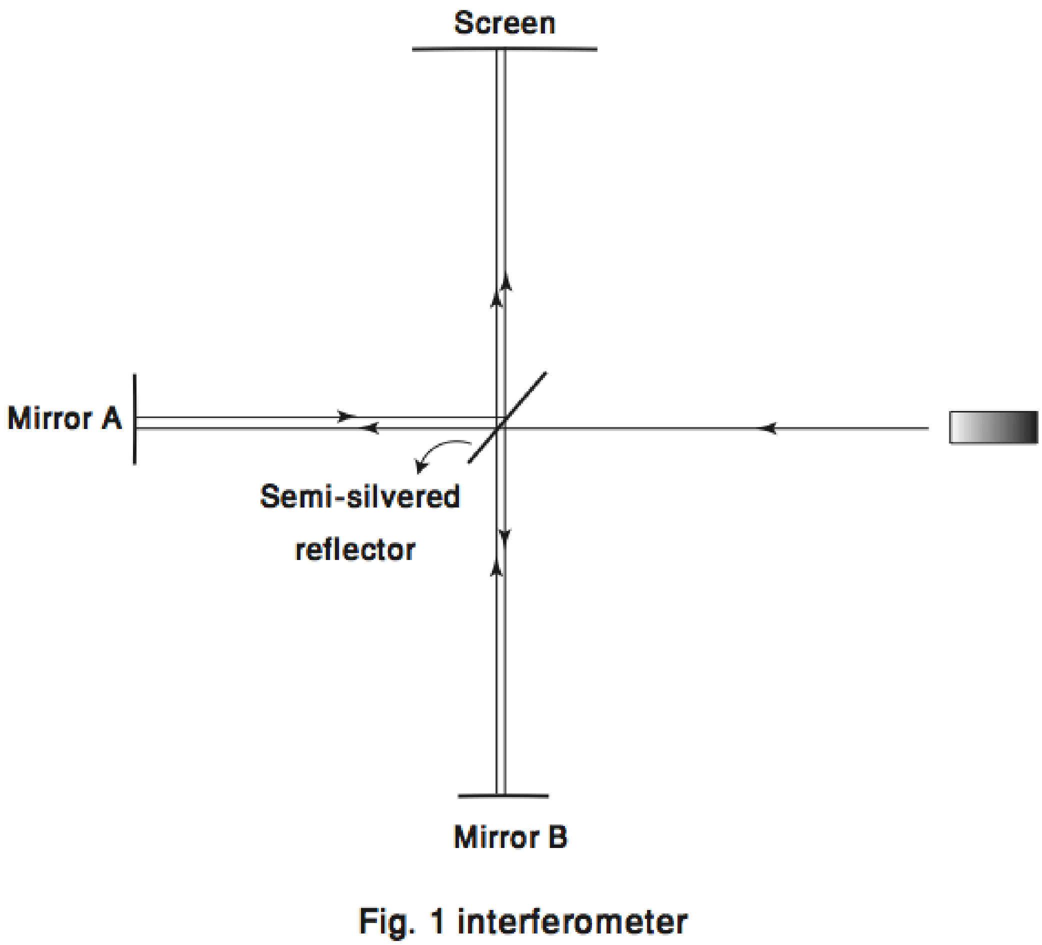

FIG. 1:

8. **Measuring Gravitational Waves**

Think of the LIGO inteferometer and suppose a gravitational wave passes through it. The gravtitaional wave has a direction and contracts one of the arm by a fraction x while lengthens the other arm by same fraction x for a short while. While this happens the carefully analyzed interferometer registers that a bright fringe has changed into a dark fringe. Supposing that the Laser being used had a wavelength of 700 nm and each arm has a length of 4 km, what is the fractional change x in lengths that the gravitational wave must have caused on its path as it passes through. The interferometer can actually detect even a billionth part of a full shift from bright to dark fringe. What is its best sensitivity in terms of fractional length change that it can measure using this laser beam?

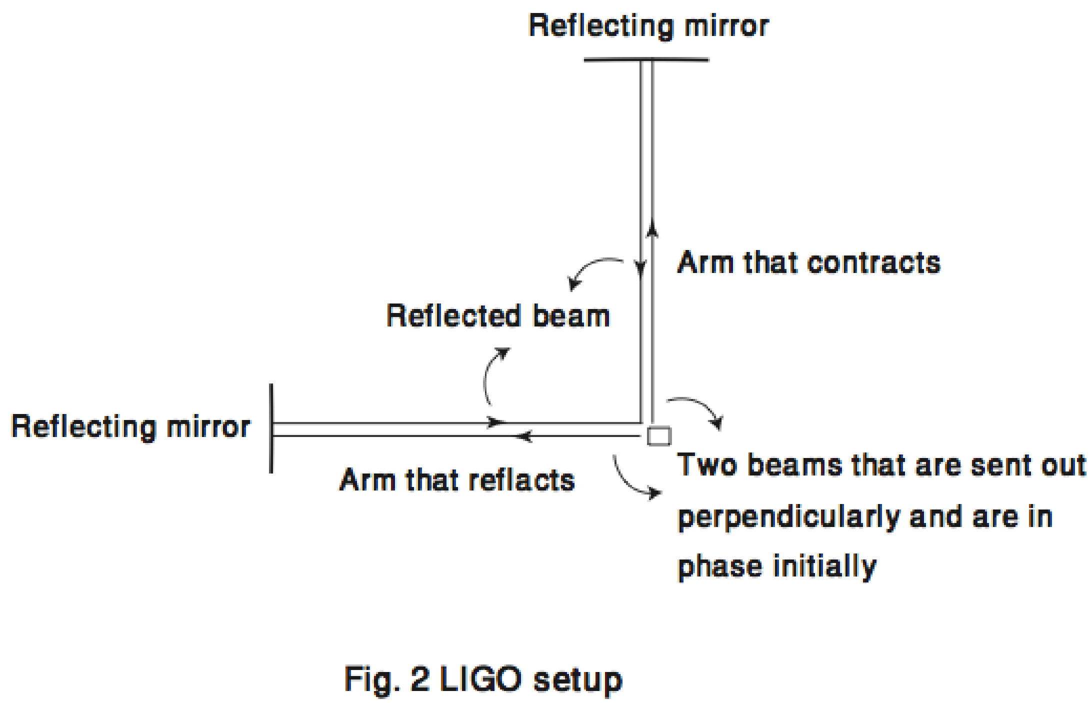
Reflecting mirror
Arm that contracts
Reflected beam
Reflecting mirror
Arm that reflacts
Two beams that are sent out perpendicularly and are in phase initially
Fig. 2 LIGO setup

FIG. 2: