Problem Set 3: Bohr's Atom

1. A photon is emitted as an atom makes a transition from $n = 4$ to $n = 2$ level. What is the frequency, wavelength and energy of the emitted photon?

2. For the Balmer series i.e., the atomic transitions where final state of the electron is $n = 2$, what is the longest and shortest wavelength possible? Is any of the frequency of Lyman series, which corresponds to transitions where electron ends up in $n = 1$ level, in the visible region?

3. A Hydrogen atom initially in its ground state i.e., $n = 1$ level, absorbs a photon and ends up in $n = 4$ level. What must have been the frequency of the photon? Now the electron makes spontaneous emission and comes back to the ground state. What are the possible frequencies of the photons emitted during this process.

4. **Non-relativistic Nature of Electron Inside Bohr's Atom**

   Using Bohrs quantization rule, derive a formula for electrons speed in the quantized Bohrs orbits. By putting in the values of constants, explicitly derive the value of electrons speed in $n = 1$ orbit. What fraction of speed of light it is? Is it justified to treat electrons motion as non-relativistic in Hydrogen atom as we have been doing? What happens to the speed in higher levels?

5. **An Atom with Anti-Electron in the Center**

   Go back to the derivation of Bohrs formula for quantized orbits. In reality, the electron is not merely moving around the proton, rather the two particles are moving around the center of mass and we can reduce the problem to that of a single particle (of reduced mass) moving around this center of mass. This is what you did in Mechanics course to solve two body central force problem. Explain why did then we put mass of electron in all our formulas instead of reduced mass. Positronium is an atom made up of a positive electron (a positron, the anti-particle of electron) and a usual negative electron moving around each other. For this atom, find the Bohrs radius and first few frequencies of "Balmer" series, i.e. when this atom makes transitions to $n = 2$ orbit from $n = 3, 4, 5$ orbits. Compare the frequencies with Hydrogen atom.

6. **Patching Bohr's Model with Larger Everyday World**

We saw that in class how we can mesh up the fact of photon nature of light, that interaction of light with matter is always in terms of a discrete bundles of energy interacting with matter, with the smooth flow of radiation we are used to of seeing in everyday life, and which is characteristic of Maxwells theory, when the total energies involved are so large that we tend to ignore the small discrete packets. Let us see how this works out in case of Hydrogen atom. Just like an electron oscillating with a given frequency emits radiations of same frequency, Maxwells theory predicts that an electron orbiting in a circle with a certain frequency of revolution will emit light of the same frequency. As it does so, it loses energy and its orbit shortens, increasing the frequency of revolution and in next instant it emits radiation of a bit larger frequency in a continues fashion. Since we can test all of this for energies of larger scales, this picture should come out from our quantum formulas in those limits.

(a) Show that in Bohrs theory, the frequency of photon emitted as the electron makes a transition from an orbit with quantum number n to an orbit of quantum number $n-1$ is given by,

$$f = \frac{m_e e^4}{8\epsilon_o^2 h^3}\left[\frac{2n-1}{(n-1)^2 n^2}\right]$$

For large orbits, as we encounter in everyday situations,n is very large. Take that limit in the above formula, ignoring additions of order 1 to the large number n.

(b) Now calculate frequency of revolution of the electron moving in an orbit quantized according to Bohrs formula (put Bohrs formula for electrons radius in n-th orbit) and show that it comes out exactly equal to the frequency of photon derived in part a.

(c) Explain in detail what would one observe (about the radiation) if an electron were revolving around a proton in a macroscopic sized radius and how it matches with Maxwells predictions. Do we need to use quantum theory for motion of particles in macroscopic orbits?

(d) Find the frequency of photons emitted by an electron moving around a proton in a radius of 1 cm.

7. **A Quantum Nano Solar Cell**

   A nano-scale P-N junction has only 100 atoms in its depletion region with each capable of producing only one electron-hole pair. In other words there are only 100 electrons available capable of jumping from valance level (band) to the conduction level. To start, all the electrons are in valence level and hence there is no free electron or hole so no conduction occurs. When a light of suitable frequency and a certain given intensity is shone on this junction continuously, it is found that there are 15 electrons promoted to conduction level at any given time, which are then swept away by the internal field producing a current.

   (a) Recalling the facts which we discussed in class regarding the interaction of light with electrons, argue that if we double the intensity, the number of free electrons (electrons in conduction level) will not be doubled but would be less than that.

   (b) Show that no matter how much light we shine, we will never be able to promote more than 50 electrons to conduction level in a steady state.

8. **Laser Characteristics**

   Recall our laser set-up from the class with energy levels 1, 2 and 3. Level 2 is the highest and Level 3 has energy in between levels 1 and 2. Level 2 is very short lived and spontaneously goes down to level 3 with much more probability than going to level 1. Level 3 on the other hand is long lived and obviously electron there can only go to level 1 by spontaneous emission. Photons of energy $E_{12} = E_2 - E_1$ are used to pump electrons and achieve population inversion between level 1 and 3.

   For each of the following changes, describe what would change in output laser characteristics in terms of its frequency and power and what changes would be required in the light that is being used for pumping in terms of its frequency and power.

   (a) Both levels 2 and 3 are raised by same fixed amount with respect to level 1. All probabilities remaining same.

   (b) Level 2 remains same while level 3 is brought closer to the level 1, all probabilities remaining same.

   (c) Level 3 remains same while level 2 is raised up with respect to level 1, all probabilities remaining same.

(d) All levels stay at the same level but the probability of level 2 going to level 3 by spontaneous emission is reduced (but still higher) compared to it going to level 1.

(e) All else remains same but the probability of level 2 going to 1 by spontaneous emission becomes more than it going to level 3.

(f) All else remains same but life time of level 3 is reduced.

9. Pair creation is a phenomenon in which a photon is converted to a pair of particle and antiparticle, both of exactly same mass. The energy of photon is converted into energy of electrons (notice that even at rest, the electrons have energy). On the other hand in pair annihilation, the particle anti-particle pair annihilate each other, are vanished, and their energy is converted into giving off a photon.

   (a) Consider a photon of angular frequency ω which is converted to a particle anti-particle pair at a height h above the ground. The particles are produced in rest with having no energy due to motion. What would be the mass of each particle in terms of ω?

   (b) Now the two particles fall down the height h. What is the total energy possessed by the particles downstairs? Down there, they annihilate each other and their energy is converted to give off a photon. What is the frequency of this photon, in terms of ω and h?

   (c) Now we send this newly created photon back to height h. In order to avoid the creation of energy, this photon must lose energy so that at height h it just have enough energy to create the pair of same mass again, without having any extra energy. What must be this energy loss for this to happen? What is the change in frequency in climbing up the height h? Does it agree with the formula we derived in class?

10. **Gravitational Red Shift**

   (a) A monochromatic laser light of blue color with a wavelength of exactly 450 nm is flashed directly upwards from ground floor. Someone on the 15th floor, at a height of 50 m receives it. What wavelength they would find this light to have?

(b) A Neutron star is a very compact object with mass equal to about the mass of the sun compressed within a radius of 10 km. They are routinely observed as pulsars in the sky. Suppose the same experiment as in part a is repeated from the surface of a Neutron star. What would be the wavelength now at a height of 50 m? Would it induce a noticeable color change?