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Adv. Q.M. Leture 2

$S = \int L\, dt$ $\quad \delta S = 0 \Rightarrow$ eqns of motion

$H(q,p) = p\dot{q} - L(q,\dot{q})$

$\dot{A} = \{A, H\}$ $\quad \{A,H\} \rightarrow \frac{-i}{\hbar}[A,H]$

$\downarrow$

Quantization.

$\Rightarrow$ H is the generator of time translation both in C.M + Q.M.

$\Rightarrow$ Take H of C.M + use it in Q.M. to generate dynamics (in any of the pictures).

Maxwell's Equations:

No monopole.

$\vec{\nabla} \cdot \vec{E} = 4\pi\rho$, $\quad \vec{\nabla} \cdot \vec{B} = 0$

Gauss's law

$\vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} = 0$, $\quad \vec{\nabla} \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\vec{J}$, Consider $\vec{\nabla} \cdot \vec{B} = 0$

This equation can be trivially satisfied if we write.

$$\boxed{\vec{B} = \vec{\nabla} \times \vec{A}}$$

then $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0.$

put in Faraday's law:-

$$\vec{\nabla} \times \vec{E} + \frac{1}{c} \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{A}) = 0$$

$$\Rightarrow \vec{\nabla} \times \left(\vec{E} + \frac{1}{c} \frac{\partial \vec{A}}{\partial t} \right) = 0$$

$$\Downarrow$$

$$-\vec{\nabla} \Phi$$

$$\Rightarrow \boxed{\vec{E} = -\vec{\nabla} \Phi - \frac{1}{c} \frac{\partial \vec{A}}{\partial t}}$$

Two Maxwell's eqn's are already satisfied.

$\Rightarrow$ Put in Gauss's law

$$\boxed{-\nabla^2 \Phi - \frac{1}{c} \frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{A} = 4\pi\rho} \text{ — Gauss's law}$$