Lecture 3:-

Last time we established the Hamiltonian in arbitrary E.M. Fields i.e.,

$$H(p,q) = \frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2 + e\Phi$$

where

$$\vec{p} = m\dot{\vec{q}} + \frac{e}{c}\vec{A}$$

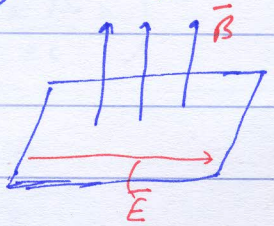
* Note that mechanical momentum (physical in classical case) is

$$m\dot{\vec{q}} = m\vec{v} = \vec{p} - \frac{e}{c}\vec{A}$$

Let's give it a name: $\vec{\pi}$.

Example:- Magnetic Field piercing through a thin slab

This is ~~quantum~~ Hall set up:-



A choice of $\vec{A}$ which gives this field is

$$\vec{A} = \left(-y\,\hat{i} + x\,\hat{j}\right)\frac{B_0}{2} \Rightarrow A_x = -\frac{yB_0}{2},\quad A_y = \frac{B_0}{2}x.$$

$$\vec{B} = \vec{\nabla}\times\vec{A} = B_0\,\hat{j}.$$