Lecture 5 :-

(Ahranov & Bohm Effect).

Let us first recall an easy example from elemantary quantum Mechanics: A particle on a ring.

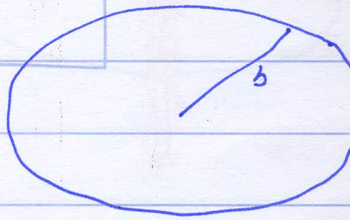
Since we will be using cylinderical symmetries, lets write down the main differential operators involved in cylinderical co-ordinates:

$$\vec{\nabla} f = \hat{r}\frac{\partial}{\partial r} f + \hat{\varphi}\frac{1}{r}\frac{\partial}{\partial \varphi} f + \hat{z}\frac{\partial f}{\partial z}.$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial}{\partial \varphi}A_\varphi + \frac{\partial}{\partial z}A_z.$$

$$\vec{\nabla}^2 = \frac{1}{r}\left(\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right) + \frac{1}{r}\frac{\partial}{\partial \varphi}\left(\frac{1}{r}\frac{\partial}{\partial \varphi}\right) + \frac{\partial^2}{\partial z^2}.$$

Consider a particle confined to move on a circle of radius "b". The Hamiltonian is



$$-\frac{\hbar^2}{2mb^2}\frac{\partial^2}{\partial \varphi^2}\psi(\varphi) = E\,\psi(\varphi)$$. Energy e.v. eqn.

$$\Rightarrow \frac{\partial^2}{\partial \varphi^2}\psi(\varphi) = -\frac{2mb^2}{\hbar^2}E\,\psi(\varphi)$$

call $\frac{2mb^2}{\hbar^2}E = k^2 \Rightarrow E = \frac{\hbar^2 k^2}{2m\,b^2}$.

Solutions are

$\psi(\varphi) = e^{ik\varphi}$ with continuity condition

$$\psi(\varphi + 2\pi) = \psi(\varphi)$$

$$\Rightarrow \quad e^{ik(\varphi + 2n\pi)} = e^{ik\varphi}$$

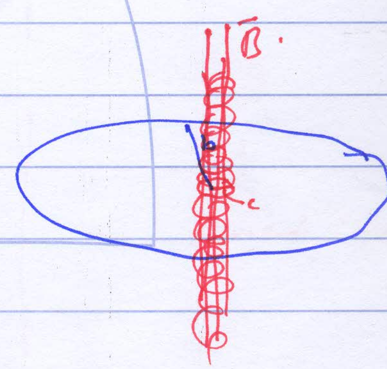
$$e^{ik2\pi} = 1$$

$$\Rightarrow \quad \Rightarrow \quad \boxed{k = \pm n.}$$

$$\Rightarrow \quad E = \frac{\hbar^2 n^2}{2mb^2} \qquad n = 0, \pm 1, \pm 2$$

Energy is 2-fold degenerate.

Now put a coil through the ring carrying flux Φ.
radius of coil : c.
Field Strength inside coil = $\vec{B}$.
flux = $\boxed{\Phi = B\pi c^2}$



We consider a long solenoid so that all the field is confined within the solenoid; there is no field at the location of electron.
What is $\vec{A}$.

We have using cylindrical symmetry (inside the coil)

$$\int_S \vec{B} \cdot \vec{ds} = \Phi_{in} = B\pi r^2 \text{ for a circle inside the coil.}$$

$$\int (\vec{\nabla} \times \vec{A}) \cdot d\vec{a} = \oint \vec{A} \cdot \vec{dr} = B\pi r^2$$

$$A_\varphi (2\pi r) = B\pi r^2$$

$$\Rightarrow \quad A_\varphi = \frac{Br}{2} \qquad \Rightarrow \quad \vec{A} = \frac{Br}{2} \hat{\varphi}.$$