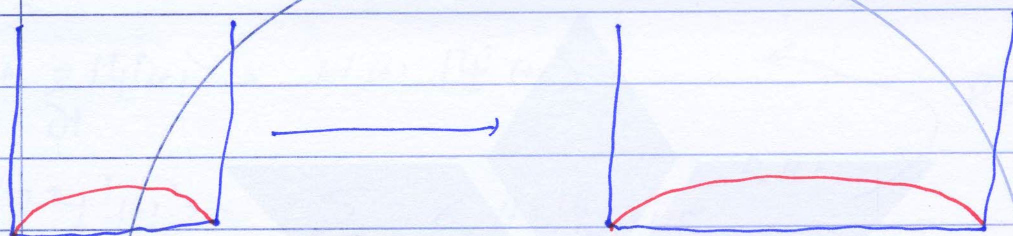
LECTURE 7:-

ADIABATIC THEOREM & BERRY'S Phase.

~~For Small slow changes in a parameter of a Hamiltonian, wave function maintains its energy eigenstates as time progresses~~

If a system is in an energy eigenstate of a Hamiltonian and we change the Hamiltonian slowly by varying a parameter slowly, the system remains in that eigen state of the new Hamiltonian.



We have

$$H(t)\,\psi_n(t) = E_n(t)\,\psi_n(t)$$

at $t=0$, $\quad H(0)\,\psi_n(0) = E_n(0)\,\psi_n(0)$

as $t \to \infty \quad H(t)\,\psi_n(t) = E_n(t)\,\psi_n(t)$

where $E_n(t)$ is the n-th eigen state of the Hamiltonian

Proof:-

Notice that in the absence of time dependence in parameters an eigen state evolves as

$$\Psi_n(t) = \psi_n e^{-iE_n t/\hbar}$$

Now we still have

$$H(t)|\psi_n(t)\rangle = E_n(t)|\psi_n(t)\rangle$$

with $\langle\psi_m(t)|\psi_n(t)\rangle = \delta_{nm}$. at one "t".

We have for

$$i\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle = H(t)|\Psi(t)\rangle$$

Put here

when $$|\Psi(t)\rangle = \sum_n c_n(t)|\psi_n(t)\rangle e^{i\theta_n(t)}$$

Since $\psi_n(t)$ are still complete at any time, the co-efficients of course change arbitrarily and $e^{i\theta_n(t)}$ has been taken out for later convinience.

$$\theta_n(t) \equiv -\frac{1}{\hbar}\int_0^t E_n(t')\,dt'$$

Schrodinger Equation gives me

$$i\hbar\left[\sum_n \dot{c}_n(t)|\psi_n(t)\rangle e^{i\theta_n(t)} + \sum_n c_n(t)|\dot{\psi}_n(t)\rangle e^{i\theta_n(t)} + i c_n\dot{\theta}_n(t)|\psi_n(t)\rangle e^{i\theta_n(t)}\right]$$

$$= \sum_n c_n H(t)|\psi_n(t)\rangle e^{i\theta_n(t)}$$