Problem Set 1 Adv. Q.M.

Gauge Invariance of S.E.

Problem 1 As claimed in class, prove that if we make a gauge transformation in ~~S.E.~~ the E.M. potentials appearing in S.E., the solution to new equation is given by gauge transformation of the wavefunction i.e.

If
$$\frac{1}{2m}\left[\left(-i\hbar\vec{\nabla}-q\vec{A}\right)^2\right]\psi(x,t)+V(x,t)\psi(x,t)$$
$$=i\hbar\frac{\partial}{\partial t}\psi(x,t)$$

then $\psi'(x,t)=e^{-i\frac{q\Lambda(x,t)}{\hbar}}\psi(x,t)$ satisfies:

$$\frac{1}{2m}\left[-i\hbar\vec{\nabla}-q\left(\vec{A}-\vec{\nabla}\Lambda(x,t)\right)\right]^2\psi'(x,t)+\left(V(x,t)+\frac{\partial\Lambda}{\partial t}\right)\psi'(x,t)$$
$$=i\hbar\frac{\partial}{\partial t}\psi'(x,t)$$

Problem 2: Operator Ordering in S.E.

We obtained our S.E. by using the replacement

$$\frac{1}{2m}\left(\vec{P}-q\vec{A}\right)\left(\vec{P}-q\vec{A}\right)=\frac{1}{2m}\vec{P}^2+q^2\vec{A}^2-q\vec{P}\cdot\vec{A}-q\vec{A}\cdot\vec{P}$$ and

then putting $\vec{P}=-i\hbar\vec{\nabla}$.