Recitation 1

1. (a) Draw the following waves on top of each other, on the same axis.

$$\begin{aligned} f_1(x) &= A\sin(kx) \\ f_2(x) &= A\sin\left(kx + \frac{\pi}{2}\right) \\ f_3(x) &= A\sin\left(kx - \frac{\pi}{2}\right) \\ f_4(x) &= A\cos(kx + \pi). \end{aligned}$$

   (b) Draw the resultant wave that is obtained by adding (superposing) f_1 and f_2.

2. Consider two waves

$$\begin{aligned} f_1(x,t) &= A\sin(k_1 x - \omega_1 t) \\ f_2(x,t) &= A\sin(k_2 x - \omega_2 t), \end{aligned}$$

   where $k_2 = k_1 + \Delta$, where Δ is very small. i.e., the wavelengths of the two waves are very close to each other. By superposing the two waves, find a mathematical expression for the resultant wave that would allow you to draw a snapshot of the wave accurately. Draw the snapshot. This is the phenomena of beats.

3. Consider the wave equation that we discussed in the class

$$\frac{\partial^2 f}{\partial t^2} = c^2 \frac{\partial^2 f}{\partial x^2}$$

   where this time we consider the quantity that is waving to be complex function of space and time. i.e., $f(x,t)$ gives us a complex number for each position x in space at a given time t.

   (a) Show that $f(x,t) = Ae^{i(kx-\omega t)}$ is a solution. What is the condition on ω and k for this to be a solution?

   (b) What is the "wavelength" and "time period" of this solution? What is the speed?

4. Consider another wave equation

$$i\frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t),$$

$\hbar$ and m, for us are just some constants for now.

Show that $\psi(x,t) = Ae^{i(kx-\omega t)}$ are a solution of this equation, what is the relation between k and ω for this to be a solution?