Problem Set 2: Photons

1. Consider the double slit set up that we considered in the class and recall its main features. A coherent but faint source of light is shone on the double slit apparatus from left. On Screen S, where light is finally detected, we have detectors available capable of measuring extremely small intensities of light. We observe that only ashes of light are detected, always of same energy if we are using a single frequency light, which we interpret as particles of light i.e., photons. When we collect the ashes, count and plot them, we get the interference pattern. On the other hand, if one hole is closed, we get a pattern that is just peaked in front of the open hole as shown in figure. If we open the holes alternately, and wait for some time to collect the ashes, we just get the sum of two such peaks and no interference. All this persists, even if the light is so faint that we record ashes at a very slow rate such that there is only one photon crossing the apparatus on average. Also, if we put detectors near the holes, these detectors also only detect a ash of same size, only at one of the detector at a time. In short, you can not divide the size of the ash. Remember, the pattern on the screen is formed only after collecting many ashes (or particles of light i.e., photons).

   Try to explain these features, using the four possibilities for a picture of the dual particle and wave nature of light that were discussed in class. Explain which one works and which one fails and why. If all of them fail, try to come up with a picture of your own which could work.

   Answer 1:

2. Consider a source of light that is capable of giving of light of single wavelength. Suppose It is emitting light of wavelength $\lambda_1 = 1.40 \times 10^{-7}$ m with a power of 0.005 Watts.

   (a) How many photons are being emitted by the source in one second?

   (b) Now suppose you double the wavelength but keep the power same. How many photons are being emitted now?

   (c) When the light with original wavelength λ_1 is incident on a metal , some electrons are emitted. The electrons in the metal need an unknown amount ϕ of energy , at the very least, just to be liberated from the metal surface. Suppose it is found that the highest kinetic energy of the emitted electrons is 4 eV . Now suppose you again double the wavelength and shine this light on the same metal surface. What would be the maximum kinetic energy of the emitted electrons this time?

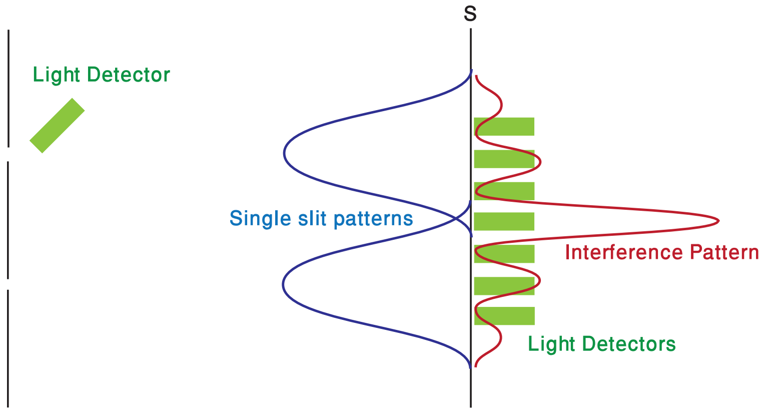


FIG. 1: Double Slit Experiment

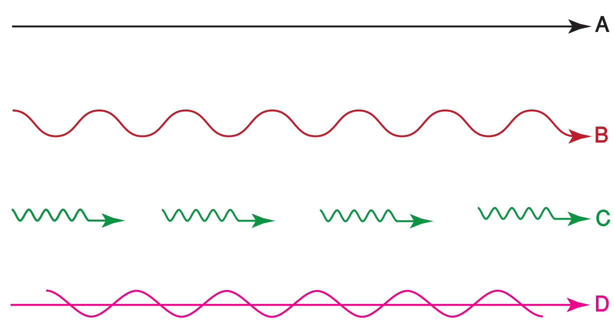


FIG.2: Photon vizualization

(d) Now you take the same material but halve the wavelength of the light being used. What would be the maximum kinetic energy of the emitted electrons now?

Answer 2**:**

(a) Let us first compute the energy carried by a photon whose wavelength is 1.40×10^{-7} m.

$$E_1 = \frac{hc}{\lambda_1} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{1.4 \times 10^{-7}} = 1.42 \times 10^{-18} \text{ J}.$$

The light beam has power equal to 0.005 watts, which means that in each second it delvers energy of 0.005 J. Now we can find the number of photons being emitted by the source in one second as:

$$N_1 = \frac{E_{\text{light}}}{E_1} = \frac{5 \times 10^{-3}}{1.42 \times 10^{-18}} = 3.52 \times 10^{15}.$$

(b) In this case the wavelength of light is $\lambda_2 = 2\lambda_1 = 2.8 \times 10^{-7}$ m. So the energy of each photon is:

$$E_2 = \frac{hc}{\lambda_2} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{2.8 \times 10^{-7}} = 7.1 \times 10^{-19} \text{ J}.$$

So the number of photon is:

$$N_2 = \frac{E_{\text{light}}}{E_2} = \frac{5 \times 10^{-3}}{7.1 \times 10^{-19}} = 7.04 \times 10^{15}.$$

(c) When a light beam strikes a metal such that the metal emits electrons, the energy of light is used in two processes:

i. Emitting the electron out of the metal.

ii. Providing kinetic energy to the free electrons.

So we can write the following energy conservation equation.

$$E_{\text{photon}} = \phi + K.E_e, \tag{1}$$

where ϕ is the amount of energy that is needed to knock electrons out of the metal.

In the given problem, energy of a photon is $E_{\text{photon}} = hc/\lambda_1 = E_1$ and the maximum kinetic energy gained by a free electron is 4 eV $= 6.4 \times 10^{-19}$ J. (Recall 1 eV $= 1.6 \times 10^{-19}$ J. Hence we can find the value of ϕ.

$$\phi = E_{\text{photon}} - K.E_e = (1.42 \times 10^{-18}) - (6.4 \times 10^{-19}) = 7.8 \times 10^{-19} \text{ J}.$$

Now we want to find the value of maximum K.E when the wavelength of the photon is λ_2 and hence its energy is E_2. We know the value of ϕ so we can just plug values in equation (**??**):

$$K.E = E_2 - \phi = (7.1 \times 10^{-19}) - (7.8 \times 10^{-19}) = -0.7 \times 10^{-19} \text{ J}.$$

In this case, kinetic energy is negative because $E_2 < \phi$. This means that the energy of a photon is less than the minimum energy required to knock an electron out of the metal. Hence no electrons are emitted when a light beam of wavelength λ_2 is used.

(d) Let λ_3 be the wavelength of new light, then

$$\lambda_3 = \frac{\lambda_1}{2} = \frac{1.4 \times 10^{-7}}{2} = 7 \times 10^{-8} \text{ m}.$$

We can find energy of a photon in this light beam:

$$E_3 = \frac{hc}{\lambda_3} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{7 \times 10^{-8}} = 2.84 \times 10^{-18} \text{ J}.$$

Then the maximum kinetic energy of the emitted electrons is:

$$K.E_e = E_3 - \phi = (2.84 \times 10^{-18}) - (7.8 \times 10^{-19}) = 2.06 \times 10^{-18} \text{ J}.$$

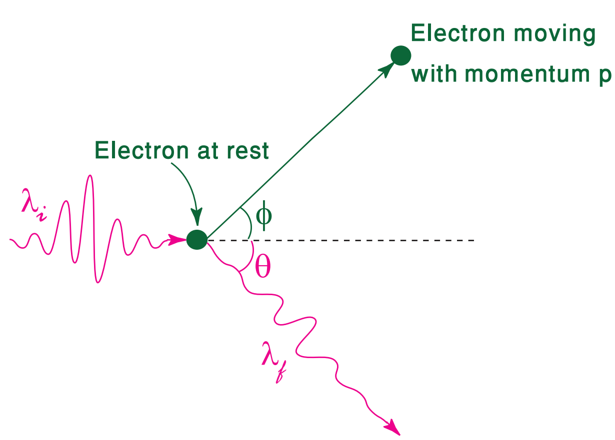
3. Complete the derivation of recitation problem 3 to derive that the shift in wavelength in Compton effect is given by

$$\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_0 c}(1 - \cos\theta),$$

where m_0 is electron's rest mass, c is velocity of light and λ_f, λ_i are wavelengths of scattered and incident X-rays respectively. If the light always strike the electrons like billiard balls, then this shift should occur whenever light interacts with electrons (radio signal transmission, light reflecting from a surface). Why did people not notice this shift in wavelength of light before Compton and how come they believe Maxwell's theory which predicts no shift in wavelength.

Answer 3:

Compton effect involves a light beam of wavelength λ_i that strikes with an electron at rest, and gets scattered such that its wavelength changes to λ_f. The following figure gives a good depiction of this process.



We want to show that the difference between wavelengths of scattered and incident photons has following expression:

$$\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_0 c}(1 - \cos\theta), \tag{2}$$

where m_0 is the rest mass of the electron and θ is the angle shown in figure.

We know that in the collision, the energy of the system as well as both horizontal and vertical components of the momentum are conserved. So we can write down the equations that conserve energy and momentum and see if they help us derive equation (??).

First, consider the energy conservation equation. Initially, the photon has wavelength λ_i so it has energy $hc/\lambda_i = p_i c$, where $p_i c$ is the momentum of the photon. Also, we should incorporate the rest energy of electron, which is $m_0 c^2$. After collision, the photon has energy $hc/\lambda_f = p_f c$ and the electron has energy $\sqrt{(pc)^2 + (m_0 c^2)^2}$ since it has non zero momentum p now.

$$\begin{aligned} p_i c + m_0 c^2 &= p_f c + \sqrt{(pc)^2 + (m_0 c^2)^2} \\ (p_i - p_f)c + m_0 c^2 &= \sqrt{(pc)^2 + (m_0 c^2)^2}. \end{aligned} \tag{3}$$

Now consider the vertical components of the momentum. Initially, the photon is moving in horizontal direction so the vertical component of its momentum is zero. After the collision, the vertical components of momenta of the photon and the electron are $-p_f \sin\theta$ and $p \sin\phi$ as evident from the figure.

$$\begin{aligned} 0 &= -p_f \sin\theta + p \sin\phi \\ p \sin\phi &= p_f \sin\theta \end{aligned} \tag{4}$$

For the horizontal component of momentum, the initial value is p_i which is the momentum of incident photon. After the collision, the components of momenta are $p_f \cos\theta$ and $p \cos\phi$, so we have the following equation:

$$\begin{aligned} p_i &= p \cos\phi + p_f \cos\theta \\ p \cos\phi &= p_i - p_f \cos\theta. \end{aligned} \tag{5}$$

We want to solve for $\lambda_f - \lambda_i$, so we should keep p_f and p_i but eliminate p from the equations above. To eliminate p, we should square two equations (??) and (??), add them and plug the value of p^2 in equation (??).

Squaring equation (**??**) gives:

$$p^2 \sin^2 \phi = p_f^2 \sin^2 \theta. \tag{6}$$

Squaring equation (**??**) gives:

$$p^2 \cos^2 \phi = p_i^2 + p_f^2 \cos^2 \theta - 2p_i p_f \cos \theta. \tag{7}$$

Adding equation (**??**) and (**??**) gives:

$$\begin{aligned} p^2 \sin^2 \phi + p^2 \cos^2 \phi &= p_f^2 \sin^2 \theta + p_i^2 + p_f^2 \cos^2 \theta - 2p_i p_f \cos \theta \\ p^2(\sin^2 \phi + \cos^2 \phi) &= p_i^2 + p_f^2(\sin^2 \theta + \cos^2 \theta) - 2p_i p_f \cos \theta \\ p^2 &= p_i^2 + p_f^2 - 2p_i p_f \cos \theta. \end{aligned} \tag{8}$$

Now we can plug this value in equation (**??**). We get:

$$(p_i - p_f)c + m_0 c^2 = \sqrt{(p_i^2 + p_f^2 - 2p_i p_f \cos \theta)c^2 + m_0^2 c^4}.$$

Squaring both sides gives:

$$\begin{aligned} (p_i - p_f)^2 c^2 + m_0^2 c^4 + 2m_0 c^3 (p_i - p_f) &= (p_i^2 + p_f^2 - 2p_i p_f \cos \theta)c^2 + m_0^2 c^4 \\ p_i^2 + p_f^2 - 2p_i p_f + m_0^2 c^2 + 2m_0 c(p_i - p_f) &= p_i^2 + p_f^2 - 2p_i p_f \cos \theta + m_0^2 c^2 \\ -2p_i p_f + 2m_0 c(p_i - p_f) &= -2p_i p_f \cos \theta \\ -p_i p_f + m_0 c(p_i - p_f) &= -p_i p_f \cos \theta \\ m_0 c(p_i - p_f) &= p_i p_f (1 - \cos \theta) \\ \frac{(p_i - p_f)}{p_i p_f} &= \frac{(1 - \cos \theta)}{m_0 c}. \end{aligned}$$

Recall that momentum of a photon and wavelength of an electromagnetic wave are related through $p = h/\lambda$. Now plug this in equation above.

$$\begin{aligned} \frac{h/\lambda_i - h/\lambda_f}{h^2/\lambda_i \lambda_f} &= \frac{(1 - \cos \theta)}{m_0 c} \\ \frac{\lambda_f - \lambda_i}{h} &= \frac{(1 - \cos \theta)}{m_0 c} \\ \lambda_f - \lambda_i &= \frac{h}{m_0 c}(1 - \cos \theta). \end{aligned}$$

Which is the required expression for change in wavelength.

4. The energy gap between valance and conduction band in a material is 2.5 eV. What is the minimum frequency of light that you must shine to make a solar cell made out of this material to work?

Answer 4:

To make a solar cell, we want to provide enough energy to the electrons in valence band such that they move to the conduction band and contribute to providing current in the material. The energy gap between valence band and conduction band for material in the question is 2.5 eV, so we must provide at least 2.5 eV energy through light so that the current starts to flow.

$$E_{\text{min}} = 2.5 \text{ eV} = 2.5 \times 1.6 \times 10^{-19} \text{ J} = 4 \times 10^{-19} \text{ J}.$$

To find the minimum value of frequency, we should use $E = hf$:

$$\begin{aligned} hf_{\text{min}} &= 4 \times 10^{-19} \text{ J} \\ f_{\text{min}} &= \frac{4 \times 10^{-19}}{6.626 \times 10^{-34}} \\ &= 6 \times 10^{14} \text{ Hz}. \end{aligned}$$

5. A light of wavelength $\lambda = 750$ nm is shone on a solar cell with a power of 1 mW falling on a square centimeter of the cell. We see that a current of 50 mA runs through the circuit as a result of it. It is then found that if we double the power of light the current also doubles, assuring us of a linear response of the current to the intensity of light. Estimate how much current will run if we shine a light of $\lambda = 400$ nm on this cell, again with a power of 1 mW falling on a square centimeter of the cell.

Answer 5:

As the light being shone on the solar cell has wavelength $\lambda = 7.5 \times 10^{-7}$ m, the energy of a photon is:

$$E_1 = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{7.5 \times 10^{-7}} = 2.65 \times 10^{-19} \text{ J}.$$

We know that power delivered by light on one square centimeter of the cell is 1 mW $=$ 10^{-3} W. This means that light transfers 10^{-3} J of energy to 1 cm^2 of the cell in 1 second.

Using this we can find the number of photons n that are transferred to 1 cm^2 of the sollar cell in 1 second:

$$n = \frac{E_{\text{light}}}{E_{\text{1photon}}} = \frac{10^{-3}}{2.65 \times 10^{-19}} = 3.8 \times 10^{16} \text{ photons per second.}$$

Due to this light, 50 mA $= 5 \times 10^{-2}$ A current flows in the circuit. We know that doubling the power of light doubles the amount of current. This happens because doubling the power of light doubles the number of incident photons which doubles the number of electrons that now contribute to current in the circuit. So we have a direct proportionality between number of photons n and the value of current I:

$$I \propto n.$$

We should introduce a constant of proportionality p:

$$I = pn.$$

We can find the value of p usibg $I = 5 \times 10^{-2}$ A and $n = 3.8 \times 10^{16}$.

$$p = \frac{I}{n} = \frac{5 \times 10^{-2}}{3.8 \times 10^{6}} = 1.3 \times 10^{-17} \text{ C.}$$

Now we let the wavelength of light vary. The new wavelength is 4×10^{-7} m. So while power remains the same, the number of photons should vary.

$$\begin{aligned} E_{\text{1photon}} &= \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{4 \times 10^{-7}} = 5 \times 10^{-19} \text{ J} \\ n &= \frac{E_{\text{light}}}{E_{\text{1photon}}} = \frac{10^{-3}}{5 \times 10^{-19}} = 2 \times 10^{15} \text{ photons per second.} \end{aligned}$$

Now the value of current can be found using $I = pn$:

$$I = 1.3 \times 10^{-7} \times 2 \times 10^{5} = 2.6 \times 10^{-2} \text{ A} = 26 \text{ mA.}$$

6. A $P - N$ junction solar cell can run on light of maximum wavelength of 450 nm. We provide it a forward bias and there is a current owing through it. As the electrons reach the P-type material, Or rather midway in the depletion region, they start to annihilate the holes by filling them up. What minimum frequency of light will come out as a result of this current conduction? Can these solar cells be used as sources of light of desired frequency?

Answer 6:

The solar cell can run on maximum wavelength of $\lambda = 450$ nm $= 4.5 \times 10^{-7}$ m. This value of maximum wavelength corresponds to the following value of minimum energy required to run the solar call.

$$E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{4.5 \times 10^{-7}} = 4.42 \times 10^{-19} \text{ J} = 2.76 \text{ eV}. \tag{9}$$

This is the minimum energy required to help an electron transition from valence band to the conduction band of the semiconductor.

Now, if we run the circuit in forward bias, without shining any light on the P-N junction, we see that as an electron reaches the depletion region and annihilates a hole, it releases energy. The corresponding picture in band diagram is that an electron jumps from conduction band to the valence band. The energy that the electron releases in this process is the "band gap" which is the same as the amount of energy we calculated in equation (**??**).

To find the value of minimum frequency we use value of band gap energy.

$$f = \frac{E}{h} = \frac{4.42 \times 10^{-19}}{6.626 \times 10^{-34}} = 6.67 \times 10^{14} \text{ Hz}.$$

This is the minimum frequency of the light that will come out as a result of conduction.

7. For a long time in medical history, X-rays have been considered to be a necessary evil, though they are being replaced by other imaging techniques now. Explain why they were necessary and why they were evil? Why light of longer wavelengths like visible light or radio waves could not be used for imaging? Why their evil character can not be done away by softening the beam?

Answer 7:

X-rays are necessary as they are electromagnetic waves with small wavelengths. Small wavelengths can be used to prob structures at microscopic level, as seen in the case of Bragg diffraction.

They are evil, because according to $E = hc/\lambda$, smaller wavelength corresponds to higher energy and higher energy photons can cause breaking of bonds between molecules at the cellular and subcellular level.

The reason we cannot use lights of longer wavelengths is that they cannot probe very small distances at the cellular level. Similarly, we cannot soften the X-ray beam because lower energy will correspond to higher wavelengths, which is not useful for imaging processes.

8. It is said that one should not get more than 100 exposures of normal intensity X-rays in life time or one carries the risk of developing cancer. On the other hand we are exposed to sunlight all the time. Which exposure carries more energy? Why sunlight is not dangerous? It is claimed by many people that cell phone signals can cause cancer. What is your educated guess about it based on what you have learnt about light so far. Remember, cell phone signals work in radio frequencies.

   Answer 8**:**

   We have seen in class that according to the particle picture of light proposed by Einstein, energy of a photon is directly proportional to the frequency of light waves. This is true for all electromagnetic waves. Energy of a photon in an EM wave is directly proportional to the frequency of the EM wave. Now, EM wave can be divided broadly into three categories based on the value of their frequency: infrared, visible and ultraviolet. X-rays lie in the ultraviolet region where frequency is bigger than that of waves in the visible light region. Hence, energy of an X-ray photon is larger than that of a light photon, and therefore exposure to X-rays can cause more harm to our cells as compared to the sunlight.

   As compared to the two of these, cell phone signals carry least energy as they work in radio frequencies which lie in the infrared region of the spectrum of electromagnetic waves.

9. X-rays of wavelength 0.200 nm are scattered from a block of carbon. If the scattered radiation is detected at $90°$ to the incident beam, find the shift in the beam's wavelength and also find the kinetic energy imparted to the recoiling electron. At what angle we will find the beam with greatest shift in wavelength?

   Answer 9**:**

   Whenever you hear about scattering of X-rays off a material, the first thing that should come to your mind is "Compton effect". In the present case, an X-ray beam of wavelength $\lambda_i = 0.2$ nm $= 2 \times 10^{-10}$ m strikes a block of carbon. The X-rays get scattered to various directions, in each of which there is a different shift in wavelength which we can find using the formula we derived in question 3. For the particular case of $\theta = 90°$, the shift in wavelength is:

$$\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{m_0 c}(1 - \cos 90°) = \frac{h}{m_0 c} = \frac{6.626 \times 10^{-34}}{9.109 \times 10^{-31} \times 3 \times 10^8} = 2.4 \times 10^{-12} \text{ m.}$$

The kinetic energy of the recoiled electron can be found by calculating difference between energies of incoming and outgoing photons.

$$K.E_e = E_i - E_f = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_f},$$

Since $\Delta\lambda = \lambda_f - \lambda_i, \quad \Rightarrow \lambda_f = \lambda_i + \Delta\lambda$. Thus kinetic energy will be,

$$\begin{aligned} K.E_e &= \frac{hc}{\lambda_i} - \frac{hc}{\lambda_i + \Delta\lambda} = hc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_i + \Delta\lambda}\right) \\ &= 6.626 \times 10^{-34} \times 3 \times 10^8 \left(\frac{1}{2 \times 10^{-10}} - \frac{1}{(2 \times 10^{-10}) + (2.4 \times 10^{-12})}\right) \\ &= 1.18 \times 10^{-17} \text{ J.} \end{aligned}$$

We will find the beam with greatest shift $\Delta\lambda$ where the value of $(1-\cos\theta)$ is maximum. At $\theta = 180°$, $\cos\theta = -1$ and $(1 - \cos\theta) = 2$. This is the maximum value $(1 - \cos\theta)$ can have. So we will have the beam with maximum wavelength shift at $\theta = 180°$.

10. Photons of wavelength 0.0711 nm are bombarded on a crystal. What is the wavelength of backscattered photons, the ones that come right back? What is the energy of these photons? What is the energy of the recoiled electron?

Answer 10:

Photons that come right back are the ones that are scattered at angle $\theta 180°$. The shift in wavelength of such photons is:

$$\Delta\lambda = \frac{h}{m_0 c}(1 - \cos 180°) = \frac{h}{m_0 c}(1 - (-1)) = \frac{2h}{m_0 c} = 4.8 \times 10^{-12} \text{ m.}$$

The energy of the photons that come right back can be found using their wavelength λ_f:

$$E_f = \frac{hc}{\lambda_f} = \frac{hc}{\lambda_i + \Delta\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{(7.11 \times 10^{-11}) + (4.8 \times 10^{-12})} = 2.62 \times 10^{-15} \text{ J.}$$

The energy of the recoiled electron is the sum of its rest energy m_0c^2 and its kinetic energy $K.E_e$, the latter of which can be found by calculating difference between E_f and E_i.

$$K.E_e = E_f - E_i = (2.8 \times 10^{-15}) - (2.62 \times 10^{-15}) = 1.76 \times 10^{-16} \text{ J}.$$

So the total energy of the recoiled electron is:

$$E_T = m_0c^2 + K.E_e = [9.109 \times 10^{-31} \times (3 \times 10^8)^2] + (1.76 \times 10^{-16}) = 8.2157 \times 10^{-14} \text{ J}.$$

11. In a Compton scattering , the scattered photon has an energy of 120 keV and the recoiled electron has energy of 40 keV . Find the wavelength of the incident photon as well as the angles at which both electron and the photon scatter relative to the initial direction of the photon.

Answer 11:

We are given that

$$\begin{aligned} E_f &= 120 \text{ keV} = 1.2 \times 10^5 \times 1.6 \times 10^{-19} \text{ J} = 1.92 \times 10^{-14} \text{ J} \\ K.E_e &= 40 \text{ keV} = 6.4 \times 10^{-15} \text{ J}. \end{aligned}$$

We can find energy E_i of incident photon:

$$\begin{aligned} K.E_e &= E_i - E_f \\ E_i &= E_f + K.E_e = (1.92 \times 10^{-14}) + (6.4 \times 10^{-15}) = 2.56 \times 10^{-14} \text{ J}. \end{aligned}$$

So the wavelength λ_i of the incident photon is:

$$\lambda_i = \frac{hc}{E_i} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{2.56 \times 10^{-14}} = 7.75 \times 10^{-12} \text{ m}.$$

We can find the angle θ at which the photon scatters using the formula in question 3, but first we need to find the wavelength λ_f of scattered photon.

$$\begin{aligned} E_f &= \frac{hc}{\lambda_f} \\ \Rightarrow \lambda_f &= \frac{hc}{E_f} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{1.92 \times 10^{-14}} = 1.03 \times 10^{-11} \text{ m}. \end{aligned}$$

Now using the formula from question 3:

$$\begin{aligned}
\lambda_f - \lambda_i &= \frac{h}{m_0 c}\big(1-\cos\theta\big) \\
\Rightarrow 1-\cos\theta &= \frac{m_0 c}{h}\big(\lambda_f - \lambda_i\big) \\
\cos\theta &= 1 - \frac{m_0 c}{h}\big(\lambda_f - \lambda_i\big) \\
\theta &= \cos^{-1}\left[1 - \frac{m_0 c}{h}\big(\lambda_f - \lambda_i\big)\right] \\
&= 92.96^\circ.
\end{aligned}$$

To find the angle ϕ at which the electron scatters, we can use conservation of vertical component of linear momentum during the collision. As pointed out in question 3, it gives us following equation:

$$\begin{aligned}
0 &= p\sin\phi - p_f\sin\theta \\
p\sin\phi &= p_f\sin\theta \\
\sin\phi &= \frac{p_f}{p}\sin\theta.
\end{aligned}$$

We can find the value of p, using kinetic energy of electron:

$$\begin{aligned}
K.E_e &= \frac{p^2}{2m_0} \\
\Rightarrow p &= \sqrt{2m_0(K.E_e)} \\
&= \sqrt{2\times 9.109\times 10^{-31}\times 6.4\times 10^{-15}} = 1.08\times 10^{-22}\ \text{Ns}.
\end{aligned}$$

We can also find the momentum p_f of scattered photon:

$$\begin{aligned}
p_f &= \frac{h}{\lambda_f} = \frac{6.626\times 10^{-34}}{1.03\times 10^{-11}} = 6.433\times 10^{-23}\ \text{Ns} \\
\sin\phi &= \frac{6.433\times 10^{-23}}{1.08\times 10^{-22}}\times\sin(92.96^\circ) = 0.5949 \\
\Rightarrow \phi &= \sin^{-1}(0.5949) \\
\phi &= 36.51^\circ.
\end{aligned}$$

12. You want to map the structure of a crystal of inter molecular spacing of 5 nm. Which one of these photons can do the best job in a diffraction experiment? photons of (a) 250 eV (b) 5 eV (c) 100 keV (a) All of them can do the job equally well as we are using individual photons which are particles.

Answer 12**:**

As mentioned in class, Bragg diffraction happens when the wavelength λ of electromagnetic waves shone on a crystal is much less than the intermolecular spacing d of the crystal. We can calculate the wavelengths corresponding to the three energy values given and see which one of these is small as compared to $d = 5 \times 10^{-9}$ m.

(a) $E = 250$ eV.

$$\lambda_a = \frac{hc}{E} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{250 \times 1.6 \times 10^{-19}} = 4.95 \times 10^{-9} \text{ m.}$$

(b) $E = 5$ eV.

$$\lambda_b = \frac{hc}{E} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{5 \times 1.6 \times 10^{-19}} = 2.47 \times 10^{-7} \text{ m.}$$

(c) $E = 100$ keV.

$$\lambda_c = \frac{hc}{E} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{100 \times 10^3 \times 1.6 \times 10^{-19}} = 1.24 \times 10^{-13} \text{ m.}$$

Clearly, $\lambda_a \sim d$, $\lambda_b \sim d$ and $\lambda_c \sim d$. So we should use X-rays of energy 100 keV to probe the structure of this crystal.